Exercise 47

Find the first and second derivatives of the function. Check to see that your answers are reasonable by comparing the graphs of f, f', and f''.

$$f(x) = 2x - 5x^{3/4}$$

Solution

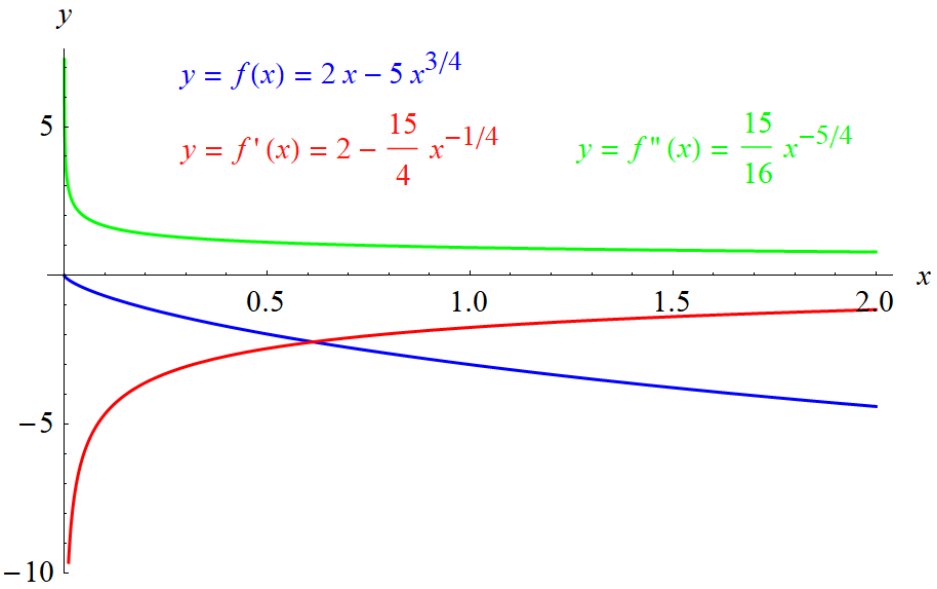
Calculate the first derivative.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(2x - 5x^{3/4}) \\ &= \frac{d}{dx}(2x) - \frac{d}{dx}(5x^{3/4}) \\ &= 2\frac{d}{dx}(x) - 5\frac{d}{dx}(x^{3/4}) \\ &= 2(1) - 5\left(\frac{3}{4}x^{-1/4}\right) \\ &= 2 - \frac{15}{4}x^{-1/4} \end{aligned}$$

Calculate the second derivative.

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left(2 - \frac{15}{4}x^{-1/4}\right) \\ &= \frac{d}{dx}(2) - \frac{d}{dx}\left(\frac{15}{4}x^{-1/4}\right) \\ &= \frac{d}{dx}(2) - \frac{15}{4}\frac{d}{dx}\left(x^{-1/4}\right) \\ &= (0) - \frac{15}{4}\left(-\frac{1}{4}x^{-5/4}\right) \\ &= \frac{15}{16}x^{-5/4} \end{aligned}$$

Below is a graph of $f(x)$ versus x, $f'(x)$ versus x, and $f''(x)$ versus x.



Since $f(x)$ is always decreasing, it makes sense that $f'(x)$ is always negative. And because $f(x)$ decreases less and less as x increases, $f'(x)$ is an increasing function. Since $f'(x)$ is an increasing function, $f''(x)$ is always positive. And because $f'(x)$ increases less and less as x increases, $f''(x)$ is a decreasing function.